Comparison of Petri Net and Markov Approach for Availability Analysis of System

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Abstract

In modern era of automation there is necessity to use high end equipment to achieve the desired production goals. Therefore, the prime task of plant engineer is to ensure availability to these equipment's in a plant for high payback ratio. Hence, availability assessment of a plant is essential in today's scenario. There are many tools and techniques for availability assessment, but Markov chains and Simulation based approach are most widely used by researchers for availability assessment. This paper presents the comparison of the two different approaches Markov approach and Petri Net approach based upon Monte Carlo simulation to analyze the availability of the system. An illustrative example is considered to support the comparison.

Keywords – Markov, Petri net, Availability, Simulation.

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Introduction

It is essential to keep the system free from failures as much as possible within given operating conditions to achieve higher production goals and long run availability of the system. In the current era of automation of systems in the industry, achieving the higher the value of plant availability is a challenging task for the plant managers. Numerous research work had been published over the years to assess the system availability using mostly analytical techniques and very few with simulation approach. (Vikas *et al.,* 2013) studied the availability modeling of a shoe upper manufacturing plant using Markov approach. They used the Runge-Kutta method to solve the probability differential equations for finding various performance parameters of a plant. (Kumar *et al.*, 2012) followed the Markov Birth-Death probabilistic approach for the formulation of a mathematical model to find the availability of a thermal power plant. (Sharma and Garg, 2011) presented the lambda-tau based mathematical modelling technique for finding the availability analysis of a fertilizer plant. (Rizwan *et al.*, 2011) discussed the reliability and availability analysis of continuous casting plant using semi Markov approach. (Garg *et al.*, 2010) used Markov approach for the performance analysis of the combed yarn plant. They have applied the Lagrange method to obtain the various state probabilities. (Gupta *et al.,* 2007) computed availability, reliability and Mean Time To Failure (MTTF) for a plastic-pipe manufacturing plant consisting of K/N units using matrix calculus method assuming the constant failure rate for different components of the system. (Cochran *et al.*, 2001) presented the availability analysis using generic Markov models. (Kumar *et al.*, 1989) studied the availability model of a washing system using a Markov model in a paper industry. (Cherry *et al.*, 1978) did the availability analysis of chemical plants using Markov chains. Besides Markov approach many researchers have used the various other tools for modelling such a reliability block diagrams, Fault trees, event tress. (Vesely *et al.*, 1971) developed a computer code for reliability and availability evaluations of the engineering systems using fault tree model. (Henley *et al.*, 1975) estimated reliability and availability parameters for process industries using reliability block diagrams. Although comparison between RBD and Markov approach had been done by (Dhillon *et al.*, 1997). But all these tools have very limited scope as the system becomes complex.

Over past years lot of research work has been done in the field of simulation modelling. Various researchers have Monte Carlo simulation-based tools and techniques for different applications. (Zhou *et al.*, 1999) discussed the applications of petri net in manufacturing systems and semiconductors manufacturing. (Murata *et al.*, 1989) discussed the basics of the petri nets. (Kleyner *et al.*, 2010) discussed the occupant safety systems using petri net. (Desrocher's *et al.*, 1995) used petri net in the field of manufacturing systems. There are very few applications of Petri net in the field of reliability and availability modelling. (Sachdeva *et al.*, 2008) used Petri net for reliability analysis of pulping system and feeding system of a paper plant. It is observed from the above literature that two mainly methods used for availability modeling are Markov approach and Simulation based models.

Petri Net

Petri Net are the modelling and graphical tool applied to industrial systems for performance evaluation (Petreson *et al.*, 1981). Especially when the systems are concurrent, parallel and asynchronous the petri net act as one of the promising tools for modelling them (Murata *et al.*, 1989). Pet nets were first originated by PhD work of Dr. Carl Adam Petri's in year 1962. Graphically, they are the bipartite graphs, which set of places P, a set of transitions T and set of directed arcs A. The places P are represented by circles; transitions are represented by rectangular bar. Places and transitions are connected to each other with the help of directed arcs. The directed arcs from Places to transitions are called input arcs whereas the arcs directed from transitions to places are called output arcs. A place may embrace tokens which are represented by dots denoting the conditions holding at any given time. Places and transitions may be connected by a number of arcs which is called the multiplicity of that input (or output) arc. A small bar with a number equal to multiplicity is used to represent the multiplicity of an arc between given place to transition. An inhibitor is used to restrict the movement of token from place to transition when the number of token in the place equal to the multiplicity of input arc.

Algebraically the petri nets are defined as the five tuple as discussed below

PN= {P, T, A, W, M_0}, where

P= {P_1, P_2…., Pm} is finite set of places.

T= {t_1, t_2,…..t_n} is a finite set of transitions.

A$\subseteq (PXT) \cup (TXP)$ is a set of arcs.

W is a weight function that takes values 1,2,3….and

M_0 is the initial marking.

The marking is the state of the petri net which is defined by the number of tokens contained in each place and is denoted by vector M=(#(P_1), (# (P_2),…., (#(P_n)), where #(P_i) denotes the number of token in a given place and n denotes the number of places. If the number of token in each place is equal to the multiplicity of the arc, then current marking is said to be enabled. The firing of enabled transition causes the movement of token s from input place to output place leading to the new marking. This movement can be represented by reachability tree or state space equations. In a reachability graph a marking M_j is said to be reachable from a marking M_i, if there exists the sequence of transitions whose firing generates M_i to M_j. A reachability graph is constructed by joining the marking Mi to M_j by a directed arc. A petri net with initial marking is given by (N, M_0).

The dynamic behavior of the petri net is illustrated with the help of an example of the series parallel system. The model has been prepared using Petri Net Module of GRIF based upon certain assumptions.

Assumptions:

- No delay in the repair except availability of the repairman.
- Priority of repair is based upon first come, first serve.
- Repair is perfect i.e. system reinstated to its original state.
- Simultaneous failure can occur in the system.
- Repair and Failure rates are not dependent on each other.

To illustrate the use of Petri Net approach an example of series parallel arrangement of four components is considered as shown in the figure 1.

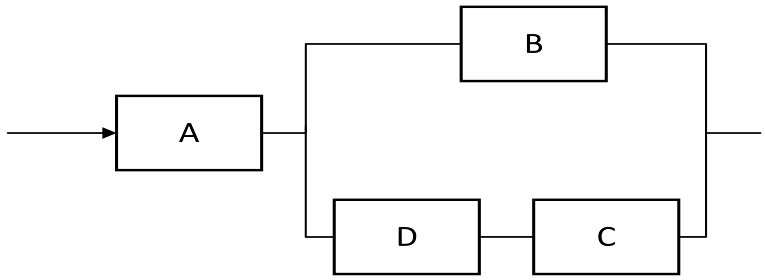

Figure 1*:* Series-Parallel system

Consider a system comprising of four components A, B, C and D arranged in series and parallel arrangement. The constant failure and repair rate of four components re •$_1$, •$_2$, •$_3$, •$_4$ and μ_1, μ_2, μ_3, μ_4 respectively. Failure of all the four components brings the system in down state, whereas if A component fails only then system will also come to down state. Failure of three components B, C and D will also lead to the system failure, whereas system will run in reduced state if either A and B are in working condition or A, D and C are in working conditions. The petri net model is shown in the figure 2.

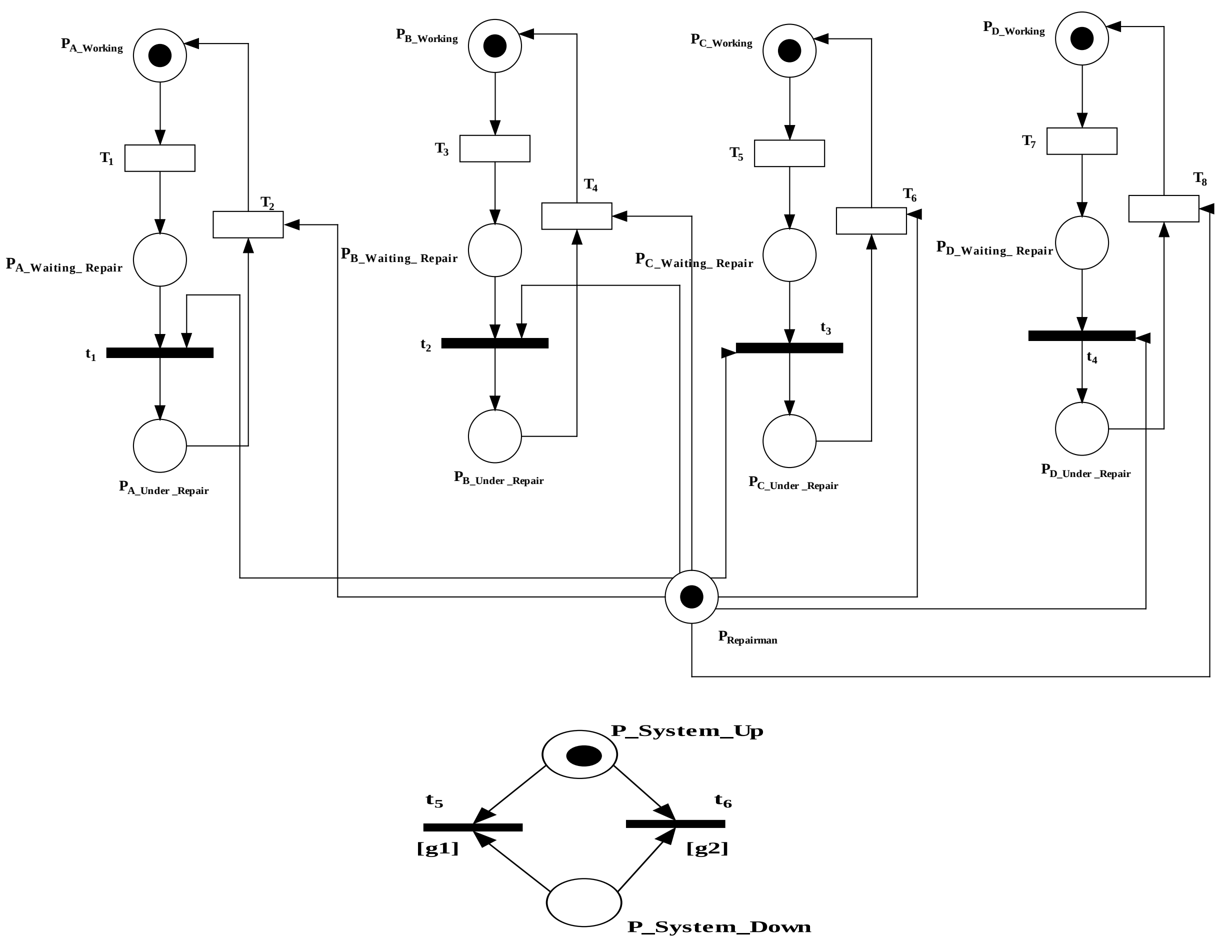

Figure 2 : Petri Net Model of System

Dynamic behavior of the system

As shown in figure 2, the transitions T_1, T_2, T_3, T_4 are having a stochastic delay related to four different components. The transition T_1 will fire the delay time has elapsed. t_1, t_2, t_3,t_4 are the immediate transitions . Initially at t=0 hrs., the transitions T1, T2, T3 and T4 are enabled and the places $P_{A\_Working}$, $P_{B\_Working,}$ $P_{C\_Working \text{ and }}$ $P_{D\_Working}$ contains token. Also, $P_{\_System\_Up}$ is marked with a token indicating the whole system is in working state and no failure has occurred yet. Let us assume T_1 is first fired the token from place $P_{A\_Working \text{ moves to}}$ $P_{A\_Wait\_Repair}$ thus enabling the guard condition at transition T_{13} to true, which results in removal of another token from place $P_{\_System\_Up}$ and placed in Place $P_{\_System\_Down.}$ The transition t_1 is now enabled as the both the places $P_{A\_Wait\_repair}$ and $P_{\_repairman}$ contains token and it fires immediately. The token is removed from places $P_{A\_Wait\_repair}$ and $P_{\_repairman}$ and is put in $P_{A\_under\ repair}$. Only T_2 is enabled now and is fired when the delay time is reached. The token disappears from $P_{A\_under\ repair}$ to $P_{A\_Working.,}$ correspondingly the guard conditions related to the T_{14} get satisfied and token disappears from $P_{\_System\_Down}$ and another appears in $P_{\_System\_Up.}$ Similarly, the dynamics of the other components will occur. The availability of the system is computed by the probability of the token in the $P_{\_System\_Up}$. The reachability graph of the petri net model discussed above is shown in Figure3.

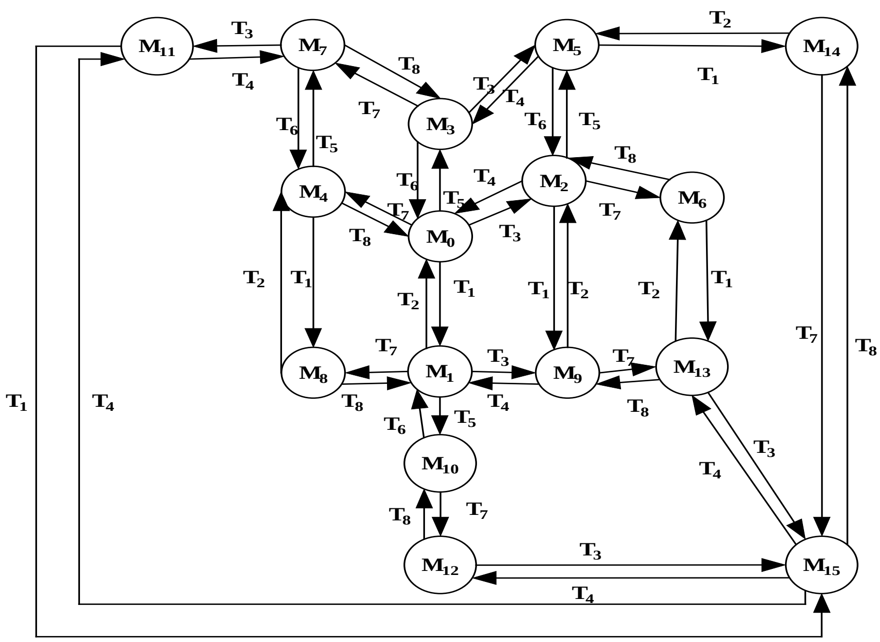

Figure 3: Reachability Graph

Markov Process

The Russian mathematician A.A. Markov introduced a special type of stochastic process in year 1907, whose present state uniquely determines the future probability, that is, with behavior of non-hereditary or memory-less. The behavior of a many of physical systems falls into this category, therefore it can be used for availability and reliability modelling of these systems. A Markovian stochastic process with a discrete state space and discrete time space is referred to as a Markov chain. If the time (index parameter) space is continuous, it is referred as the Markov process. The state transition diagram of the above illustrated example is depicted in the figure 4 with some assumptions listed below.

Assumptions

A^W, B^W, C^W, D^W : It indicates the working state of the components.

A^F, B^F, C^F, D^F : It indicates the Failed state of the components.

•$_i$: Represents the mean failure rate of the Components.

μ_i: Represents the mean repair rate of the Components.

$P_i(t)$: Probability that at time "t" all components are good and the system is in ith state.

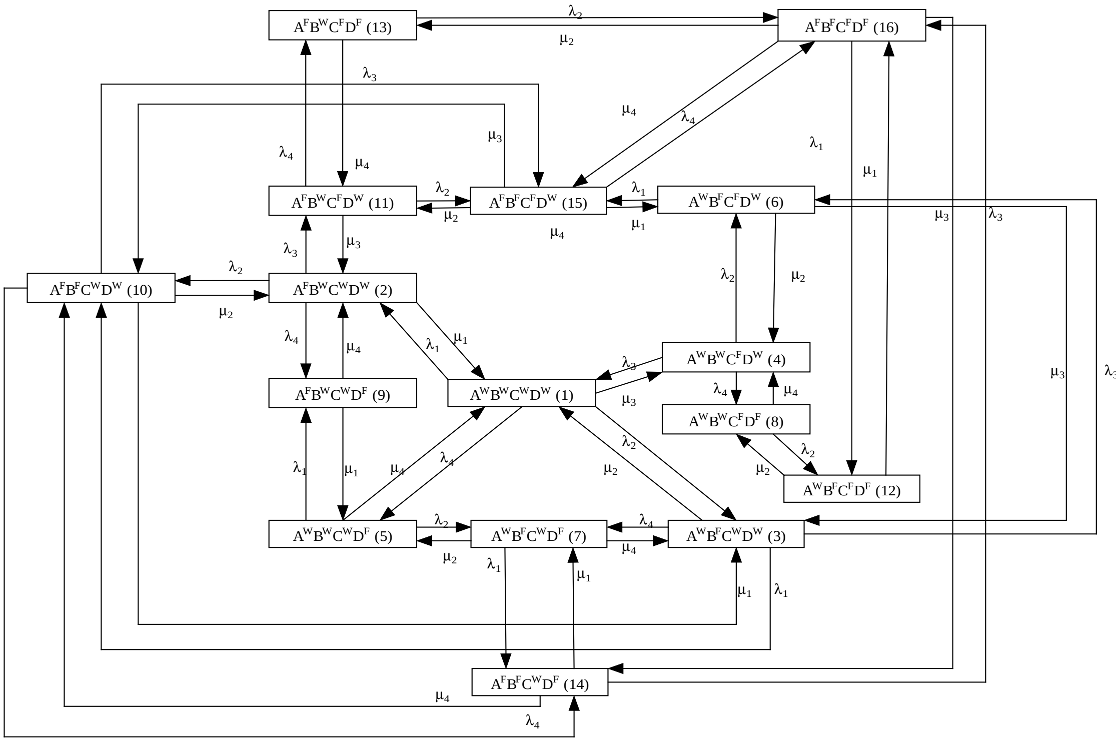

Figure 4: State transition diagram

With respect to the state transition diagram some of the first order differential equations for probability considerations are as under.

$$\frac{dP_1(t)}{dt} + (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)P_1(t) = \mu_1 P_2(t) + \mu_2 P_3(t) + \mu_3 P_4(t) + \mu_4 P_5(t) \quad (1)$$

$$\frac{dP_2(t)}{dt} + (\lambda_2 + \lambda_3 + \lambda_4)P_2(t) = \mu_2 P_{10}(t) + \mu_3 P_{11}(t) + \mu_4 P_9(t) \quad (2)$$

$$\frac{dP_3(t)}{dt} + (\lambda_1 + \lambda_3 + \lambda_4)P_3(t) = \mu_1 P_{10}(t) + \mu_3 P_6(t) + \mu_4 P_7(t) \quad (3)$$

$$\frac{dP_4(t)}{dt} + (\lambda_2 + \lambda_3 + \lambda_4)P_4(t) = \mu_2 P_6(t) + \mu_3 P_1(t) + \mu_4 P_8(t) \quad (4)$$

$$\frac{dP_5(t)}{dt} + (\lambda_1 + \lambda_2 + \lambda_3)P_5(t) = \mu_1 P_9(t) + \mu_2 P_7(t) + \mu_4 P_1(t) \quad (5)$$

$$A(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t) + P_5(t) \quad (6)$$

Similarly, another differential equation can be written for probability consideration for all other states of the diagram. These differential equations can be solved with the Laplace transformation using various numerical methods. The availability of this system can be found from eq. (6).

Reachability graph for Markov chain

The major problem faced by the reliability engineer while doing the performance analysis of a complex system is to transform the problem into Markov chain. This problem is being easily dealt with the help of reachability graph. The state evolutions of state transition diagram are represented by the markings of the reachability graph. As shown in figures 3 and 4The directed arc means the marking change of the state evolution by firing of the labeled transition, e.g., M_0 is changed to M_1 by firing T1. For a complex system, it's easy to develop the reachability graph from the petri net model of the system and further transform it into Markov chain.

It can be observed from above that the Markovian approach to finding the availability of the system needs lots of computational effort and it is often difficult to solve Markov equations analytically. Although the state transition diagrams are easy to formulate for small systems, but as the number of components of the system increases, the system more complex and size of the state transitions grows exponentially. These limitations are circumvented with Petri nets as they are easily formulated even for complex systems and one can produce the reachability graph of the petri net model.

Conclusion

There are various analytical and simulation-based methods are being used to study the various performance parameters of the system. The Markov process is one of the most widely used analytical method to study the plant performance parameters. An attempt has been made in this paper to compare the Markov and Petri Net based approach for availability assessment of complex systems. It was observed that the state space method (Markov process) is suitable for small systems. As the system grows, it is hard to transform problems into Markov chains, moreover, it is difficult to solve the Markov equations analytically. In comparison to Markov chain, it is easy to formulate the Petri net model and corresponding reachability graph even if the system gets complex. Moreover, using petri net, one can study the dynamic behavior of the system. Therefore, in case of complex systems the petri net has emerged as the powerful tool for performance modeling of the system.

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